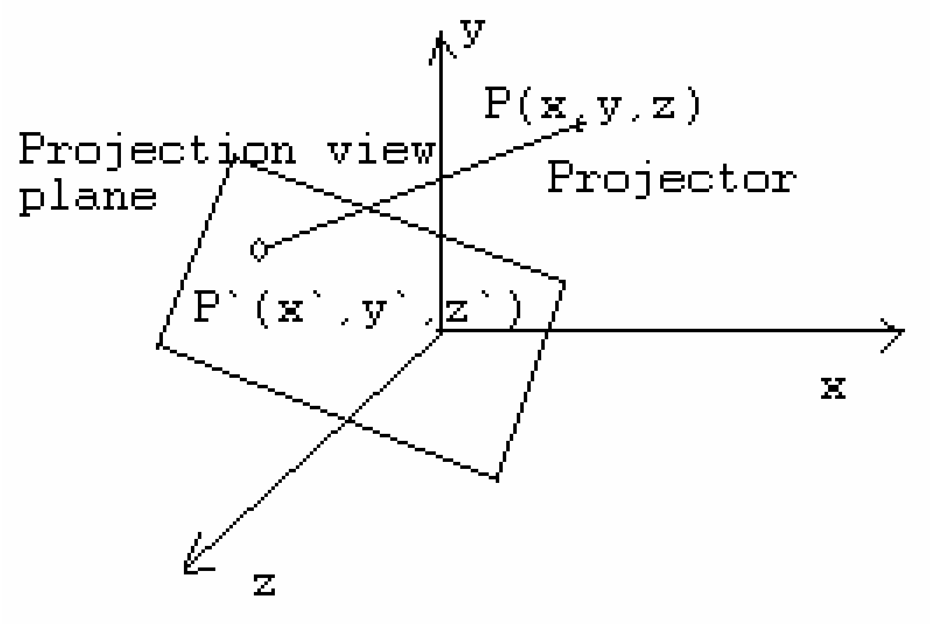
Introduction to Computer Graphics
(C S 6 0 2)
Lecture 19
Projections

For centuries, artists, engineers, designers, drafters, and architects have been facing difficulties and constraints imposed by the problem of representing a three-dimensional object or scene in a two-dimensional medium -- the problem of projection. The implementers of a computer graphics system face the same challenge.

Projection can be defined as a mapping of point P(x,y,z) onto its image P\`(x\`,y\`,z\`) in the projection plane or view plane, which constitutes the display surface. The mapping is determined by a projection line called the projector that passes through P and intersects the view plane.



The Problem of Projection

There are two basic methods of projection

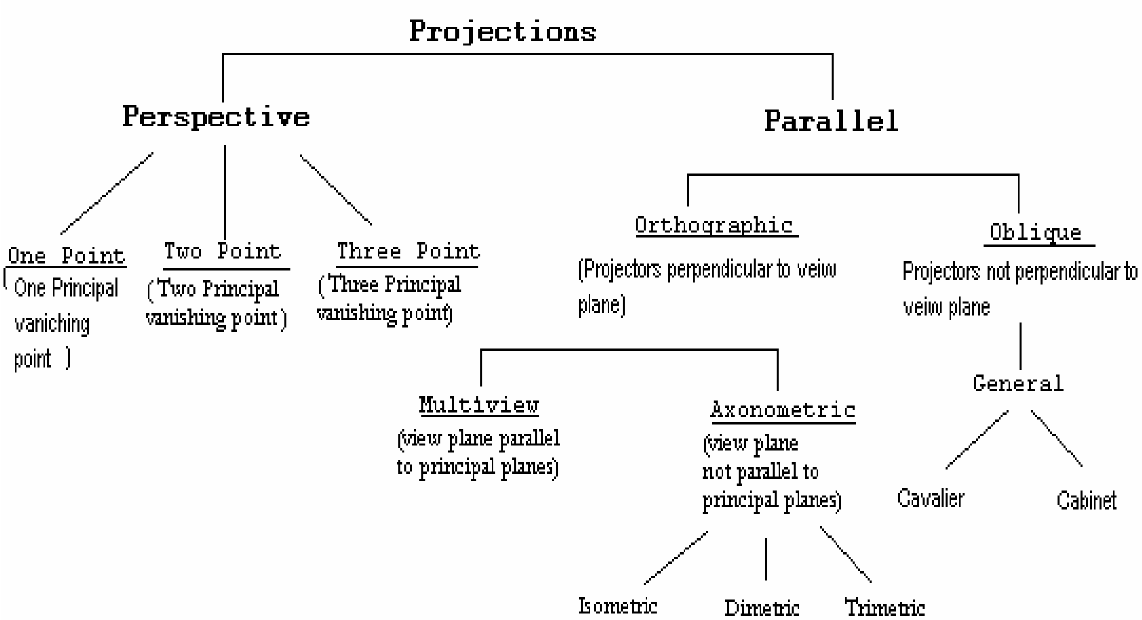
1) Parallel Projection
2) Perspective Projection

These methods are used to solve the basic problems of pictorial representations

We characterize each method and introduce the mathematical description of the projection process respectively.

Taxonomy of Projection

We can construct different projections according to the view that is desired.
Following figure provides taxonomy of the families of perspective and parallel projections. Some projections have names – cavalier, cabinet, isometric, and so on. Other projections qualify the main type of projection – one principal vanishing–point perspective and so forth.



Toxonomy of Projection

Parallel Projection

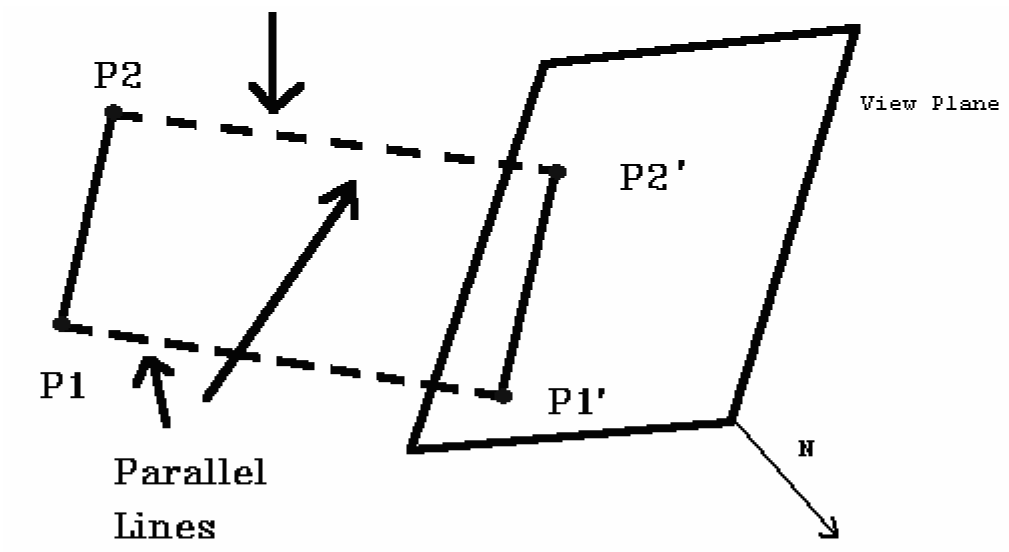
Parallel projection methods are used by drafters and engineers to create working drawings of an object which preserves its scale and shape. The complete representation of these details often requires two or more views (projections) of the object onto different view planes.

In parallel projection, image points are found as the intersection of the view plane with a projector drawn from the object point and having a fixed direction. The direction of projection is the prescribed direction for all projections. *Orthographic projections are characterized by the fact that the direction of projection is perpendicular to the view plane.* When the direction of projection is parallel to any of the principal axes, this produces the front, top, and side views of mechanical drawings (also referred to as multi view drawings).

Axonometric projections are orthographic projections in which the direction of projection is not parallel to any of the three principal axes. *Non orthographic parallel projections are called oblique parallel projection.*

Mathematical Description of a Parallel Projection

Projection rays (projectors) emanate from a Center of Projection (COP) and intersect Projection Plane (PP). The COP for parallel projectors is at infinity. The length of a line on the projection plane is the same as the "true Length".

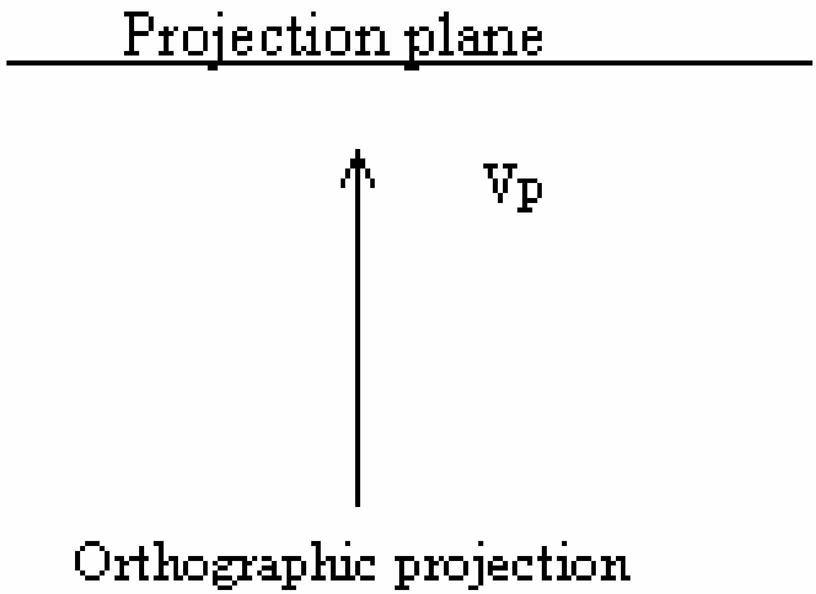


There are two different types of parallel projections:

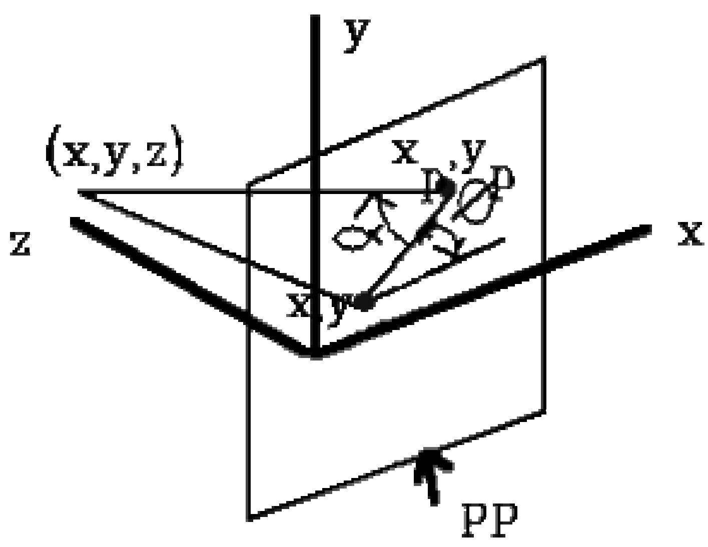
1) **Orthographic**
2) **Oblique**

1) Orthographic Projection

If the direction of projection is perpendicular to the projection plane then it is an **orthographic** projection.



Look at the parallel projection of a point (x, y, z). (Note the left handed coordinate system). The projection plane is at z = 0. x, y are the orthographic projection values and xp, yp are the oblique projection values (at angle a with the projection plane)

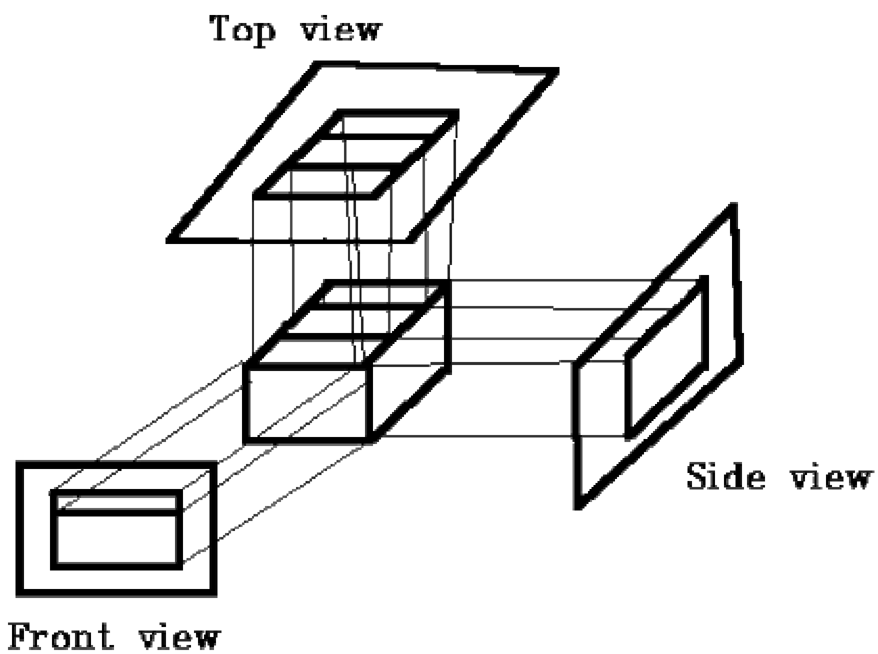


Look at orthographic projection: it is simple, just discard the z coordinates. Engineering drawings frequently use front, side, top orthographic views of an object.

Axonometric orthographic projection

Orthographic projections that show more than one side of an object are called **axonometric** orthographic projections.

Here are three orthographic views of an object.

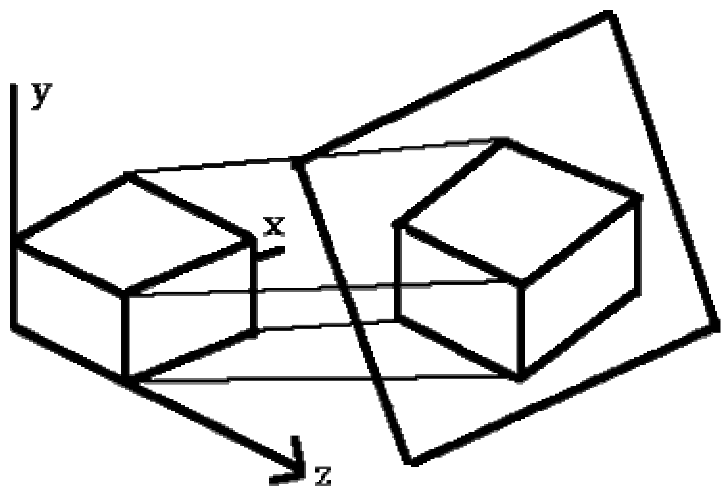


There are three axonometric projections

1) isometric
2) Dimetric
3) Trimetric

1) Isometric

The most common axonometric projection is an **isometric** projection where the projection plane intersects each coordinate axis in the model coordinate system at an equal distance or the direction of projection makes equal angles with all of the three principal axes



The projection plane intersects the x, y, z axes at equal distances and the projection plane Normal makes an equal angle with the three axes.

To form an orthographic projection xp = x, yp= y , zp = 0. To form different types e.g., Isometric, just manipulate object with 3D transformations.

2) Dimetric

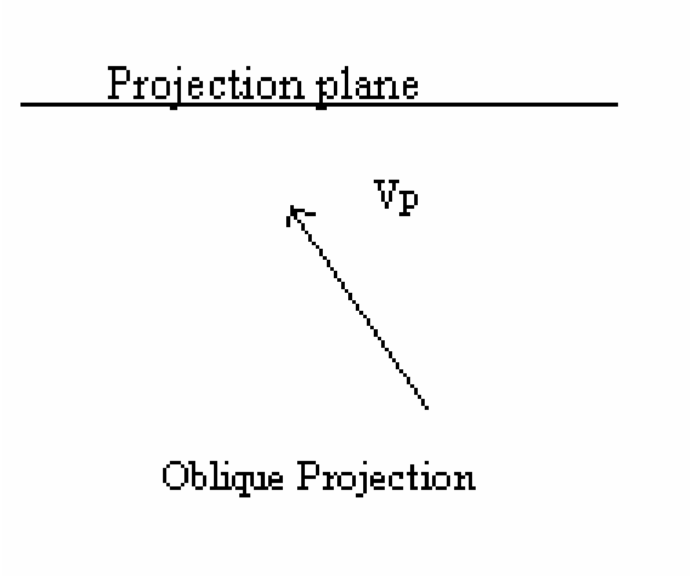
The direction of projection makes equal angles with exactly two of the principal axes

3) Trimetric

The direction of projection makes unequal angles with the three principal axes

2) Oblique Projection

If the direction of projection is not perpendicular to the projection plane then it is an **oblique** projection.



The projectors are not perpendicular to the projection plane but are parallel from the object to the projection plane.

Transformation equations for an orthographic parallel projection are straightforward. If the view plane is placed at position Zvp along the Z axis, Then any point (x,y,z) in viewing coordinates is transformed to projection coordinates as:

$$Xp = x$$

$$Yp = y$$

Where the original Z-coordinate value is preserved for the depth information needed in depth cueing and visible-surface determination procedures.

An oblique projection is obtained by projecting points along parallel lines that are not perpendicular to the projection plane. In some applications packages, an oblique projection vector is specified with two angles, alpha and phi, as shown in the figure. Point (x,y,z) is projected to position(Xp,Yp) on the view plane. Orthographic projection coordinates on the plane are (x,y). The oblique projection line from (x,y,z) to (Xp,Yp) makes an angle alpha with the line on the projection plane that joins (Xp,Yp) and (x, y). This line, of length L, is at an angle phi with the horizontal direction in the projection plane. We can express the projection coordinates in terms of x, y, L, and phi as

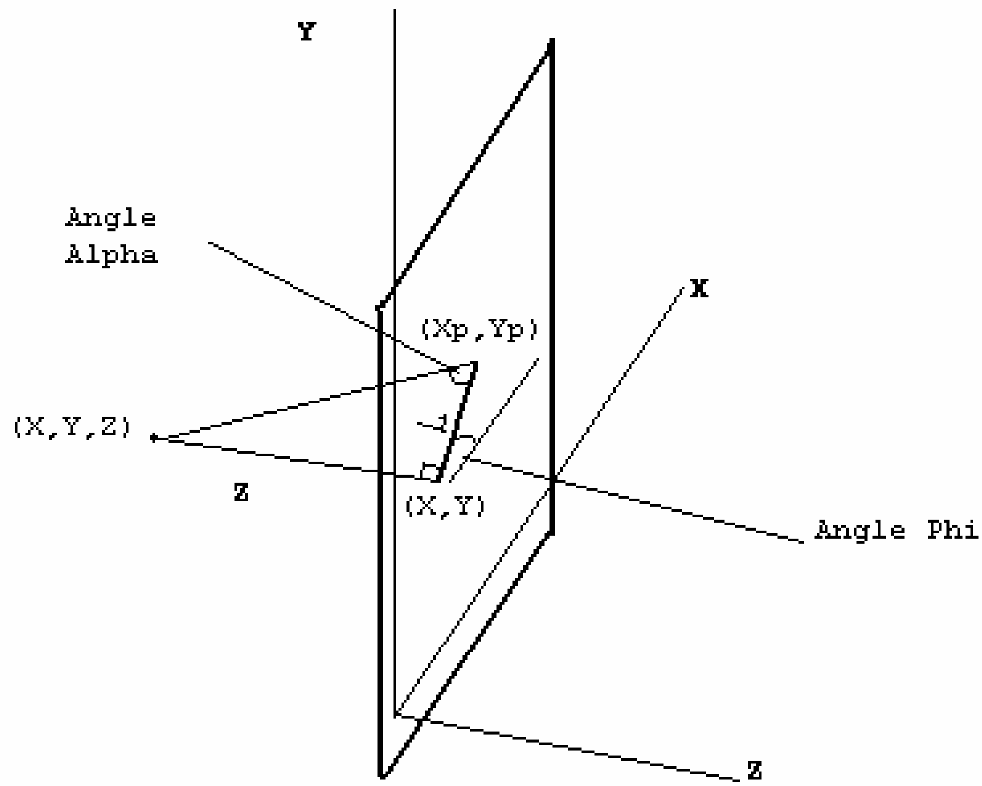


Figure: Oblique Projection of coordinate position
(x,y,z) to position (Xp,Yp) on the view plane

cos(phi) = Xp – x / L

sin(phi) = Yp – y / L

Xp = x + L cos(phi)

Yp = y + L sin(phi)

Length L depends on the angle alpha and the z coordinate of the point to be projected:

tan (alpha) = z / L

Thus,

L = z * 1/ tan (alpha)

L = z * L1

Where L1 is the inverse of tan(alpha), which is also the value of L when z = 1, we can then write the oblique projection equations.

Xp = x + z (L1 cos(phi))

Yp = y + z (L1 sin(phi))

The transformation matrix for producing any parallel projection onto the xy plane can be written as

$$\text{Parallel M} = \begin{bmatrix} 1 & 0 & L1\ \cos(phi) & 0 \\ 0 & 1 & L1\ \sin(phi) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now if Alpha = 90° (projection line is perpendicular to Projection Plane)

then

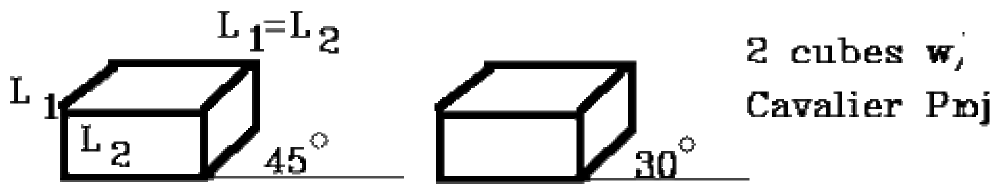
tan (Alpha) = infinity => L1 = 0, so have an orthographic projection.

Two special cases of oblique projection

1) Cavalier
2) Cabinet

1) Cavalier

Alpha = 45°, tan (Alpha) = 1 => L1 = 1 this is a **Cavalier** projection such that all lines perpendicular to the projection plane are projected with no change in length.



2) Cabinet

tan (Alpha) = 2, Alpha= 63.40°, L1 = 1 / 2

Lines which are perpendicular to the projection plane are projected at 1 / 2 length. This is a **Cabinet** projection